Assume that the lifespan of people is fixed at L years.
Assume that the birthrate is fixed at B people/yr.

If the population is at steady-state, then the total population will be B*L people.

U.S. annual birthrate is 14.14 births / 1000 people / yr
http://education.yahoo.com/reference/factbook/us/popula.html

U.S. population in 2005 is estimated at 296 Million people.
http://education.yahoo.com/reference/factbook/us/popula.html

So, B is (birthrate/100people)*(population), which is about 11,500 births/day.

$$B := 296 \times 10^{6} \cdot \frac{14.14}{1000\mathrm{yr}}$$

$$B = 1.146 \times 10^{4} \frac{1}{\mathrm{day}}$$

If (total # of people) = B*L, as we've derived from our assumptions, then L = population / B.

And if B, in turn, is (birthrate/1000people)*(population), then

$$L = \frac{\mathrm{population}}{\mathrm{population} \cdot \frac{\mathrm{birthrate}}{1000\mathrm{people} \cdot \mathrm{yr}}}$$

or

$$L = \frac{1}{\frac{\mathrm{birthrate}}{1000\mathrm{people} \cdot \mathrm{yr}}}$$

$$\frac{1}{\frac{14.14}{1000\mathrm{yr}}} = 70.721\ \mathrm{yr}$$

So, L, the fixed lifespan of people is about 71 yrs. This sounds plausible given that the estimated total lifespan at birth in 2005 was 78 yr.
http://education.yahoo.com/reference/factbook/us/popula.html

$$L := \frac{1}{\frac{14.14}{1000\mathrm{yr}}}$$

Just to run a sanity check, B*L returns our assumed population size.

$$B \cdot L = 2.96 \times 10^{8}$$

Assume that the population is well mixed -- equally likely to encounter every person in the population.

The number of people in the population with your birthdate is

$$\frac{B}{\text{population}}$$

And knowing that population is B*L, this expands to

$$\frac{B}{B \cdot L}$$

Or simply

$$\frac{1}{L}$$

$$B = 4.185 \times 10^{6} \frac{1}{\text{yr}}$$

This is also the probability that, in 1 encounter, a person will have the same birthdate as you (a match). Note that this doesn't depend on the birthrate of the the population, only people's lifespans. It is the proportion of days that are your birthdate (1 day) to the number days that people live.

$$\frac{1\,\text{day}}{L} = 3.871 \times 10^{-3}\,\%$$

We can confirm this answer by using our daily birthrate from above instead.

$$\frac{1.146 \times 10^{4}}{B \cdot L} = 3.872 \times 10^{-3}\,\%$$

What is the probability in 2 encounters of finding at least one person that shares your birthdate?

For convenience, let us assume that each encounter is independent. That is, you can encounter people you have encountered before.

Before understanding what this probability is, let us first consider what it is NOT. It is NOT:

$$2\left(\frac{1\,\text{day}}{L}\right) = 7.743 \times 10^{-3}\,\%$$

This is clear, beacuse after $\frac{L}{\text{day}} = 2.583 \times 10^{4}$ encounters, the probability of finding a match would be 100%. This cannot be true. Because we've assumed the encounters are independent, there is the possibility, however slim, that you might encounter the same non-mathcing person over and over again. From this, it's clear that you are not guarranteed to find a match, ever.

$$\frac{L}{\text{day}} \cdot \left(\frac{1\,\text{day}}{L}\right) = 100\,\%$$

The CORRECT way to calculate the probability of finding a match in 2 independent encounters (hearafter just encounters) is to consider the probability of NOT FINDING A MATCH. Which for a single encounter is simply:

$$1 - \frac{1\,\text{day}}{L} = 99.996\,\%$$

NOT finding a match in 2 ENCOUNTERS is the probability that both encounters failed. Which is

$$\left(1 - \frac{1\,\text{day}}{L}\right) \cdot \left(1 - \frac{1\,\text{day}}{L}\right) = 99.992\,\%$$

or

$$\left(1 - \frac{1\,\text{day}}{L}\right)^{2} = 99.992\,\%$$

The probability that you WILL FIND a match in 2 encounters is then:

$$1 - \left(1 - \frac{1\,\text{day}}{L}\right)^{2} = 7.743 \times 10^{-3}\,\%$$

For any number of encounters, E, the probability is

$$1 - \left(1 - \frac{1\,\mathrm{day}}{L}\right)^{E}$$

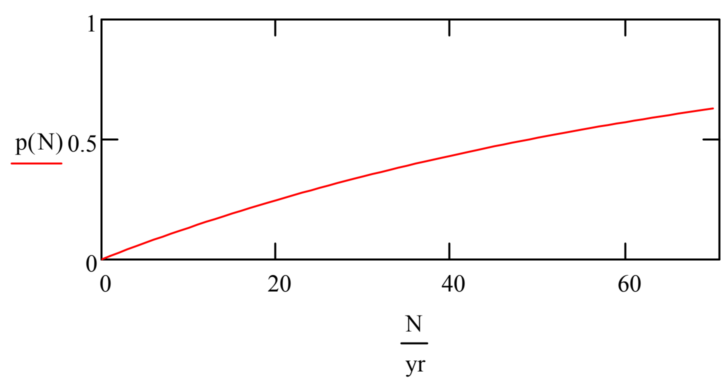
Assume that you encounter people at a fixed rate -- M.  And that the birthdate of each of these people becomes known to you.  Let's set this to 1/day... although this seems like an overestimate.  You probably only learn new people's birthdays at a rate of 1/month or less.  But let's go with 1/day for now.

$$M := \frac{1}{\mathrm{day}}$$

What is the probability of finding at least one match after N number of years?  Memento mori at L.

$$N := 0, 1\mathrm{yr}..\,L$$

$$p(N) := 1 - \left(1 - \frac{1\,\mathrm{day}}{L}\right)^{M \cdot N}$$




$$p(L) = 63.213\,\%$$

Your chance of find at least one match within your lifetime is about 63%.

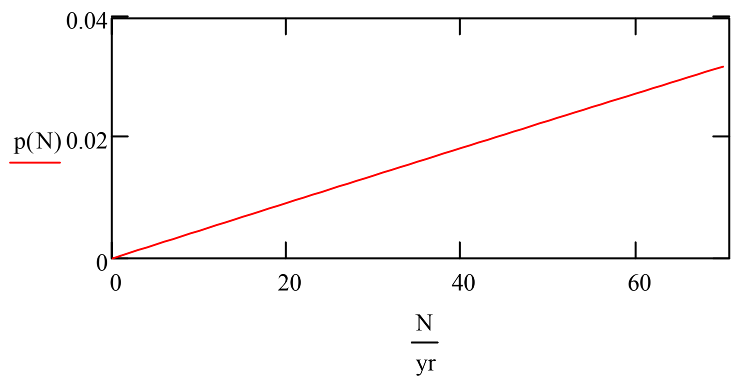
What if M is 1/month?

$$month := \frac{yr}{12}$$

$$M := \frac{1}{month}$$

$$N := 0, 1yr .. L$$

$$p(N) := 1 - \left(1 - \frac{1\,day}{L}\right)^{M \cdot N}$$




$$p(L) = 3.232\,\%$$

You have only a 3% chance of finding a match before you die.